Explicit construction of BRST charge of noncommutative D-brane system

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In the BRST-BFV scheme for noncommutative D-branes with constant NS B-field, introducing ghost degrees of freedom we construct the gauge fixed Hamiltonian and corresponding effective Lagrangian invariant under nilpotent BRST charge. It is also shown that the presence of auxiliary variables introduced via the improved Dirac formalism plays a crucial role in the construction of the BRST invariant Lagrangian.

I. INTRODUCTION

In the matrix model compactified on a torus, the three-form tensor field background has been shown to be incorporated in the supersymmetric Yang-Mills theory by deforming the base space into quantum space [1]. This implies that the D-brane field theory resides on a noncommutative space in the presence of a Neveu-Schwarz (NS) B-field background [2] due to the correspondence between the discrete light cone quantized M-theory and the D-brane world-volume field theory [3]. Relevance of the noncommutative geometry in the light cone quantization of open strings attached to D-brane has been discussed in [4]. In fact, in the matrix model the noncommutativity of spacetime is due to non-abelian Yang-Mills gauge theory induced from parallel N D-branes at coinciding positions, all on top of each other. The noncommutativity of spacetime takes place in the direction perpendicular to D-branes. On the other hand, when the end points of the open strings are attached to D-branes in the presence of a constant NS B-field, the spacetime coordinates on D-branes also do not commute in the directions parallel to D-branes. In open string theory the noncommutativity of geometry is due to the mixed boundary conditions, which are neither Neumann conditions nor Dirichlet ones. Since these boundary conditions form second-class constraints, spacetime coordinates become noncommutative. In a system with noncommutative geometry, ordinary product between functions should be replaced by the Moyal bracket [5].

The quantization of constraint systems has been extensively discussed in [6, 7]. In particular, the embedding of a second-class system into a first-class one [8], where constraints are in strong involution, has been of much interest, and has found a large number of applications [7]. Recently, in the framework of Hamiltonian analysis for the constraint systems, the noncommutativity of geometry has been studied in [9, 10]. The noncommutative D-brane system with constant B field has been also studied in the pp-wave background [11]. Moreover, noncommutative theories has been shown to be conventional theories embedded in a gravitational background produced by gauge field with a charge dependent gravitational coupling [12]. Quite recently, the noncommutative theories have been applied to varieties of models such as noncommutative Maxwell-Chern-Simons model [13] and noncommutative Yang-Mills-Chern-Simons model [14].

In the noncommutative D-brane systems with constant B field, the first-class Hamiltonian has been constructed to investigate its Dirac algebra [10]. In this paper, we will construct the first-class physical variables for this D-brane systems with the noncommutativities, and then we will introduce canonical sets of ghosts and anti-ghosts together with auxiliary fields to explicitly construct the Becchi-Rouet-Stora-Tyutin (BRST) charge, under which gauge fixed effective Lagrangian is invariant in the BRST-Batalin-Fradkin-Vilkovisky (BFV) formalism [15, 16].

II. NONCOMMUTATIVITIES IN D-BRANES WITH CONSTANT NS B-FIELD

We consider the open strings whose end points are attached at Dp-branes in the presence of a constant NS B-field. The worldsheet action of the open string of interest, to which background gauge fields are coupled, is given by

$$S = \frac{1}{4\pi\alpha'} \int_\Sigma d^2\sigma \left(g_{ij}\partial_a x^i \partial^a x^j + 2\pi\alpha' B_{ij}\epsilon^{ab}\partial_a x^i \partial_b x^j \right) + \int d\tau \left(A_i\partial_\tau x^i|_{\sigma=\pi} - A_i\partial_\tau x^i|_{\sigma=0} \right), \qquad (2.1)$$

where Σ is the string worldsheet with the metric $\eta_{ab} = \text{diag}\,(+,-)$ and the metric of target space on D-brane is taken to be Euclidean $g_{ij} = \delta_{ij}$ $(i,j = 1,2)$. Note that the action (2.1) has two $U(1)$ gauge symmetries. One of them is λ-symmetry for the transformation of $A \to A + d\lambda$ and the other is Λ-symmetry for the transformation with a form of both $B \to B + d\Lambda$ and $A \to A + \Lambda$. From the open string action (2.1) one can readily find the equations of motion to determine the mixed boundary conditions

$$g_{ij}\partial_\sigma x^j + 2\pi\alpha' \mathcal{F}_{ij}\partial_\tau x^j\big|_{\sigma=0,\pi} = 0, \tag{2.2}$$

where $\mathcal{F} = B - F$ and $F = dA$. Without background fields B and A, the boundary conditions (2.2) are Neumann boundary ones, $\partial_\sigma x^i = 0$ at $\sigma = 0,\pi$, while for $B_{ij} \to \infty$ or $g_{ij} \to 0$, the boundary conditions become Dirichlet, $\partial_\tau x^i(\tau) = 0$ on D-branes.

In order to study the boundary and the bulk near boundaries in detail, we discretize the open string action and the boundary conditions along the direction of σ with equal spacing $\epsilon = \pi/N$ which is taken to be very small with integer N, and the integral for σ being changed to the sum, $\int_0^\pi d\sigma \to \sum_{a=0}^N \epsilon$. Defining $x_{(a)}^i$ by $x_{(a)}^i(\tau) = x^i(\tau,\sigma)|_{\sigma=a\epsilon}$, we have the discretized Lagrangian of the open string action (2.1) as follows

$$\begin{aligned}
L_0 &= \frac{1}{4\pi\alpha'}\sum_{a=0}\left(\epsilon\left(\dot{x}_{(a)}^i\right)^2 - \frac{1}{\epsilon}\left(x_{(a+1)}^i - x_{(a)}^i\right)^2 + 4\pi\alpha' B_{ij}\dot{x}_{(a)}^i\left(x_{(a+1)}^j - x_{(a)}^j\right)\right) \\
&\quad + \sum_{a=0} A_i(\dot{x}_{(a+1)}^i - \dot{x}_{(a)}^i),
\end{aligned} \tag{2.3}$$

where the overdot denotes derivative with respect to τ and the mixed boundary conditions are given by

$$g_{ij}\frac{1}{\epsilon}(x_{(1)}^j - x_{(0)}^j) + 2\pi\alpha' \mathcal{F}_{ij}\dot{x}_{(0)}^j = 0, \tag{2.4}$$

with $x_{(0)}^i$ denoting the end of the open strings. Here we have taken only $\sigma = 0$ case of the boundary conditions since we are interested in the boundary $\sigma = 0$ only. Note that the action (2.3) and the boundary conditions (2.4) become continuous in σ-direction in the limit that N goes to infinity, or equivalently ϵ is very small.

From the action (2.3) we obtain the canonical momenta of $x_{(a)}^i$ $(a = 0,1,2,\cdots)$

$$p_{(a)i} = \frac{\epsilon}{2\pi\alpha'}\left(g_{ij}\dot{x}_{(a)}^j + \frac{2\pi\alpha'}{\epsilon}B_{ij}\left(x_{(a+1)}^j - x_{(a)}^j\right) - \frac{2\pi\alpha'}{\epsilon}A_i\delta_{a0}\right). \tag{2.5}$$

The combination of the boundary conditions (2.4) and the canonical momenta (2.5) gives the primary constraints [10]

$$\Omega_i = \frac{1}{\epsilon}\left((2\pi\alpha')^2\mathcal{F}_{ij}(p_{(0)}^j + A^j) + M_{ij}(x_{(1)}^j - x_{(0)}^j)\right) \approx 0, \tag{2.6}$$

where $M_{ij} = \left[g - (2\pi\alpha')^2\mathcal{F}g^{-1}B\right]_{ij}$. By taking the Legendre transformation of the Lagrangian (2.3) we can obtain the total Hamiltonian of the form $H_T = H + u^i\Omega_i$ where the canonical Hamiltonian H and the Lagrangian multipliers u^i are given by

$$\begin{aligned}
H &= \frac{1}{4\pi\alpha'\epsilon}\sum_{a=1}\left((2\pi\alpha')^2\left(p_{(a)i} - B_{ij}(x_{(a+1)}^j - x_{(a)}^j)\right)^2 + (x_{(a+1)}^i - x_{(a)}^i)^2\right) \tag{2.7} \\
&\quad + \frac{1}{4\pi\alpha'\epsilon}[\theta^{-1}\mathcal{F}^{-1}g]_{ij}(x_{(1)}^i - x_{(0)}^i)(x_{(1)}^j - x_{(0)}^j), \\
u^i &= \frac{2\pi\alpha'}{\epsilon^2\Delta_{12}}\epsilon^{ij}\left(M_{jk}p_{(1)}^k - [Mg^{-1}B]_{jk}(x_{(2)}^k - x_{(1)}^k) + \theta_{jk}^{-1}(x_{(1)}^k - x_{(0)}^k)\right), \tag{2.8}
\end{aligned}$$

with $\theta_{ij}^{-1} = -\frac{1}{(2\pi\alpha')^2}\left[(g - 2\pi\alpha'\mathcal{F})\mathcal{F}^{-1}(g + 2\pi\alpha'\mathcal{F})\right]_{ij}$. Note that with the Lagrangian multipliers (2.8) one can obtain the strongly involutive constraint algebra $\{\Omega_i, H_T\} = 0$, and the constraints (2.6) form a second-class system since their Poisson brackets are given by

$$\Delta_{ij} = \{\Omega_i, \Omega_j\} = \frac{(2\pi\alpha')^2}{\epsilon^2}\left[(G + M)g^{-1}\mathcal{F}\right]_{ij}, \tag{2.9}$$

where $G_{ij} = \left[(g + 2\pi\alpha'\mathcal{F})g^{-1}(g - 2\pi\alpha'\mathcal{F})\right]_{ij}$.

III.   FIRST-CLASS PHYSICAL VARIABLES IN D-BRANES WITH CONSTANT NS B-FIELD

Next, in order to construct the BRST invariant effective Lagrangian, we need to find the first-class physical variables, which enables us to apply the standard BRST-BFV formalism [15, 16] to the constrained systems as in unconstrained systems. Recall that the D-brane system of interest has the second-class constraints in the boundary conditions, which yields the noncommutativity of spacetime on the D-brane.

Following the Batalin-Fradkin-Tyutin scheme [7, 8], we systematically convert the second-class constraints algebra involved in the noncommutative D-brane system into a strongly involutive one by introducing a pair of canonically conjugate auxiliary variables y_i $(i = 1, 2)$ with the Poisson brackets

$$\{y_i, y_j\} = \epsilon_{ij}. \tag{3.1}$$

In this enlarged phase space one systematically constructs the first-class constraints $\tilde{\Omega}_i$ as a power series in these auxiliary variables, by requiring that they be in strong involution $\{\tilde{\Omega}_i, \tilde{\Omega}_j\} = 0$ [10],

$$\tilde{\Omega}_1 = \Omega_1 + y_1, \quad \tilde{\Omega}_2 = \Omega_2 - \Delta_{12} y_2. \tag{3.2}$$

In this extended phase we construct the first-class physical variables $\tilde{\mathcal{F}} = (\tilde{q}_a, \tilde{p}_a)$, corresponding to the original physical variables defined by $\mathcal{F} = (q_a, p_a)$. They are again obtained as a power series in the auxiliary fields (y_i, y_j) by demanding that they be in strong involution with the first-class constraints (3.2), that is $\{\tilde{\Omega}_i, \tilde{\mathcal{F}}\} = 0$. After some tedious algebra, we obtain for the first-class physical variables[1]

$$
\begin{aligned}
\tilde{x}^i_{(a)} &= x^i_{(a)} + \frac{(2\pi\alpha')^2}{\epsilon} \mathcal{F}_{12} \delta_{a0} \left(g^{i2} y_2 - g^{i1} \frac{y_1}{\Delta_{12}} \right), \\
\tilde{p}_{(a)i} &= p_{(a)i} + \frac{1}{2\epsilon} \left((G_{11} + M_{11}) \delta_{a0} - 2M_{11} \delta_{a1} \right) \left(g_{i1} y_2 + g_{i2} \frac{y_1}{\Delta_{12}} \right).
\end{aligned}
\tag{3.3}
$$

In terms of these physical variables the first-class Hamiltonian can be written in the compact form

$$
\begin{aligned}
\tilde{H} &= \frac{1}{4\pi\alpha'\epsilon} \sum_{a=1} \left((2\pi\alpha')^2 \left(\tilde{p}_{(a)i} - B_{ij} (\tilde{x}^j_{(a+1)} - \tilde{x}^j_{(a)}) \right)^2 + (\tilde{x}^i_{(a+1)} - \tilde{x}^i_{(a)})^2 \right) \\
&\quad + \frac{1}{4\pi\alpha'\epsilon} [\theta^{-1}\mathcal{F}^{-1}g]_{ij} (\tilde{x}^i_{(1)} - \tilde{x}^i_{(0)})(\tilde{x}^j_{(1)} - \tilde{x}^j_{(0)}).
\end{aligned}
\tag{3.4}
$$

We then directly rewrite this Hamiltonian in terms of the original and auxiliary variables

$$
\begin{aligned}
\tilde{H} &= H - \frac{2\pi\alpha'}{\epsilon^2} \frac{y_1}{\Delta_{12}} \left(M_{22} p^2_{(1)} - [Mg^{-1}B]_{21} (x^1_{(2)} - x^1_{(1)}) + \theta^{-1}_{21} (x^1_{(1)} - x^1_{(0)}) \right) \\
&\quad - \frac{2\pi\alpha'}{\epsilon^2} y_2 \left(M_{11} p^1_{(1)} - [Mg^{-1}B]_{12} (x^2_{(2)} - x^2_{(1)}) + \theta^{-1}_{12} (x^2_{(1)} - x^2_{(0)}) \right) \\
&\quad + \frac{\pi\alpha'}{\epsilon^3} [G + Mg^{-1}M]_{11} \left(\frac{y_1^2}{\Delta_{12}^2} + y_2^2 \right)
\end{aligned}
\tag{3.5}
$$

which is strongly involutive with the first-class constraints $\{\tilde{\Omega}_i, \tilde{H}\} = 0$.

Now, we consider the Poisson brackets of physical variables in the extended phase space $\tilde{\mathcal{F}}$ and identify the Dirac brackets by taking the vanishing limit of auxiliary variables. After some manipulation, from (3.3), one readily finds the commutators

$$
\begin{aligned}
\{\tilde{x}^i_{(a)}, \tilde{x}^j_{(b)}\} &= -(2\pi\alpha')^2 \left[(G + M)^{-1} \mathcal{F} g^{-1} \right]^{ij} \delta_{a0} \delta_{b0}, \\
\{\tilde{x}^i_{(a)}, \tilde{p}_{(b)j}\} &= \delta^i_j \left(\delta_{ab} - \frac{1}{2} \delta_{a0} \delta_{b0} \right) + \left[(G + M)^{-1} M \right]^i_j \delta_{a0} \delta_{b1}, \\
\{\tilde{p}_{(a)i}, \tilde{p}_{(b)j}\} &= \frac{1}{(2\pi\alpha')^2} \left(\frac{1}{4} \left[g\mathcal{F}^{-1}(G + M) \right]_{ij} \delta_{a0} \delta_{b0} + \left[g\mathcal{F}^{-1} M (G + M)^{-1} M \right]_{ij} \delta_{a1} \delta_{b1} \right. \\
&\quad \left. - \frac{1}{2} (g\mathcal{F}^{-1} M)_{ij} (\delta_{a0} \delta_{b1} + \delta_{a1} \delta_{b0}) \right).
\end{aligned}
\tag{3.6}
$$

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[1] In Ref. [10], the first-class variables (3.3) and their corresponding Dirac algebra (3.6) and (3.7) have not been explicitly constructed. Moreover, the ghosts and anti-ghosts, and their corresponding BRST charges, which will be discussed in the next section, have not been introduced in Ref. [10].

Note that the above Poisson brackets in the extended phase space exactly reproduce the corresponding Dirac brackets

$$
\begin{aligned}
\{\tilde{x}^i_{(a)}, \tilde{x}^j_{(b)}\} &= \{x^i_{(a)}, x^j_{(b)}\}_D, \\
\{\tilde{x}^i_{(a)}, \tilde{p}^j_{(b)}\} &= \{x^i_{(a)}, p^j_{(b)}\}_D, \\
\{\tilde{p}^i_{(a)}, \tilde{p}^j_{(b)}\} &= \{p^i_{(a)}, p^j_{(b)}\}_D,
\end{aligned}
\tag{3.7}
$$

where $\{A, B\}_D = \{A, B\} - \{A, \Omega_k\}\Delta^{kk'}\{\Omega_{k'}, B\}$ with $\Delta^{kk'}$ being the inverse of $\Delta_{kk'}$ in (2.9). Also it is amusing to see in (3.7) that these Poisson brackets of $\tilde{\mathcal{F}}$'s have exactly the same form of the Dirac brackets of the field $\mathcal{F}$ obtained by the replacement of $\mathcal{F}$ with $\tilde{\mathcal{F}}$, namely $\{\tilde{A}, \tilde{B}\} = \{A, B\}_D|_{A \to \tilde{A}, B \to \tilde{B}}$.

IV. BRST INVARIANT EFFECTIVE LAGRANGIAN

In this section, in order to obtain the BRST invariant effective Lagrangian in the framework of the BRST-BFV formalism [15, 16], which is applicable to theories with the first-class constraints discussed above, we introduce two canonical sets of ghosts and anti-ghosts together with auxiliary fields $(\mathcal{C}^i, \bar{\mathcal{P}}_i)$, $(\mathcal{P}^i, \bar{\mathcal{C}}_i)$, $(\mathcal{N}^i, \mathcal{B}_i)$, $(i = 1, 2)$ which satisfy the super-Poisson algebra

$$
\{\mathcal{C}^i, \bar{\mathcal{P}}_j\} = \{\mathcal{P}^i, \bar{\mathcal{C}}_j\} = \{\mathcal{N}^i, \mathcal{B}_j\} = \delta^i_j,
\tag{4.1}
$$

with the super-Poisson bracket defined as

$$
\{A, B\} = \frac{\delta A}{\delta q}\Big|_r \frac{\delta B}{\delta p}\Big|_l - (-1)^{\eta_A \eta_B} \frac{\delta B}{\delta q}\Big|_r \frac{\delta A}{\delta p}\Big|_l.
\tag{4.2}
$$

Here η_A denotes the number of fermions called ghost number in A and the subscript r and l imply right and left derivatives, respectively.

In the noncommutative D-brane system, we now construct the BRST charge Q and the fermionic gauge fixing function Ψ, together with the unitary gauge choice $\chi^1 = \Omega_1$, $\chi^2 = \Omega_2$,

$$
\begin{aligned}
Q &= \mathcal{C}^i \tilde{\Omega}_i + \mathcal{P}^i \mathcal{B}_i, \\
\Psi &= \bar{\mathcal{C}}_i \chi^i + \bar{\mathcal{P}}_i \mathcal{N}^i,
\end{aligned}
\tag{4.3}
\tag{4.4}
$$

with the property $Q^2 = \{Q, Q\} = 0$. This nilpotent charge Q is the generator of the following infinitesimal transformations,

$$
\begin{aligned}
&\delta_Q x^i_{(a)} = \frac{1}{\epsilon}(2\pi\alpha')^2 \delta_{a0} \mathcal{F}_{ij} \mathcal{C}^j, \\
&\delta_Q p_{(a)i} = -\frac{1}{2\epsilon}\delta_{a0}(G_{ij} - M_{ij})\mathcal{C}^j + \frac{1}{\epsilon}(\delta_{a1} - \delta_{a0})M_{ij}\mathcal{C}^j, \\
&\delta_Q y_1 = \Delta_{12}\mathcal{C}^2, && \delta_Q y_2 = \mathcal{C}^1, \\
&\delta_Q \bar{\mathcal{P}}_i = \tilde{\Omega}_i, && \delta_Q \mathcal{C}^i = 0, \\
&\delta_Q \bar{\mathcal{C}}_i = \mathcal{B}_i, && \delta_Q \mathcal{P}^i = 0, \\
&\delta_Q \mathcal{B}_i = 0, && \delta_Q \mathcal{N}^i = -\mathcal{P}^i,
\end{aligned}
\tag{4.5}
$$

under which the first-class Hamiltonian $\tilde{H}$ and $\{Q, \Psi\}$ are invariant to yield

$$
\begin{aligned}
\delta_Q \tilde{H} &= \{Q, \tilde{H}\} = 0, \\
\delta_Q \{Q, \Psi\} &= \{Q, \{Q, \Psi\}\} = 0.
\end{aligned}
\tag{4.6}
$$

The "gauge fixed" effective Hamiltonian is then given by

$$
H_{eff} = \tilde{H} - \{Q, \Psi\} = \tilde{H} - \Delta_{12}(\mathcal{C}^1 \bar{\mathcal{C}}_2 - \mathcal{C}^2 \bar{\mathcal{C}}_1) - \tilde{\Omega}_i \mathcal{N}^i - \Omega_i \mathcal{B}_i - \bar{\mathcal{P}}_i \mathcal{P}^i,
\tag{4.7}
$$

where $\tilde{H}$ is given by (2.7) and (3.5). This Hamiltonian is invariant under the transformation (4.5) with the BRST charge (4.3).

After some algebra associated with the Legendre transformation of H_{eff}, we arrive at the effective quantum Lagrangian of the form

$$
L_{eff} = L_0 + L_{WZ} + L_{ghost},
\tag{4.8}
$$

where L_0 is given by (2.3) and

$$
\begin{aligned}
L_{WZ} =\; & 2\pi\alpha' \mathcal{F}_{12}\left(-\frac{\dot{y}_1}{\Delta_{12}}\dot{x}^1_{(0)} + \dot{y}_2\dot{x}^2_{(0)} + \frac{(2\pi\alpha')^2}{2\epsilon}\mathcal{F}_{12}\left(\frac{\dot{y}_1^2}{\Delta_{12}^2} + \dot{y}_2^2\right)\right) \\
& + \frac{2\pi\alpha'}{\epsilon^2}\mathcal{F}_{12}\left(-\frac{y_1}{\Delta_{12}}(x^1_{(1)} - x^1_{(0)}) + y_2(x^2_{(1)} - x^2_{(0)}) - \frac{(2\pi\alpha')^2}{2\epsilon}\mathcal{F}_{12}\left(\frac{y_1^2}{\Delta_{12}^2} + y_2^2\right)\right) \\
& + \frac{(2\pi\alpha')^2}{\epsilon}\mathcal{F}_{12}B_{12}\left(-\frac{\dot{y}_1}{\Delta_{12}}(x^2_{(1)} - x^2_{(0)}) - y_2\dot{x}^1_{(0)} + \frac{(2\pi\alpha')^2}{\epsilon}\mathcal{F}_{12}\frac{\dot{y}_1}{\Delta_{12}}y_2\right) \\
& + \frac{(2\pi\alpha')^2}{\epsilon}\mathcal{F}_{12}B_{21}\left(\frac{y_1}{\Delta_{12}}\dot{x}^2_{(0)} + \dot{y}_2(x^1_{(1)} - x^1_{(0)}) + \frac{(2\pi\alpha')^2}{\epsilon}\mathcal{F}_{12}\frac{y_1}{\Delta_{12}}\dot{y}_2\right) \\
& + \frac{(2\pi\alpha')^2}{\epsilon}\mathcal{F}_{12}\left(\frac{\dot{y}_1}{\Delta_{12}}A_1 - \dot{y}_2 A_2\right), \\
L_{ghost} =\; & -\frac{1}{\Delta_{12}}\dot{\mathcal{B}}_2\dot{y}_1 + \dot{\bar{\mathcal{C}}}_2\dot{\mathcal{C}}^2.
\end{aligned}
\tag{4.9}
$$

This Lagrangian is invariant under the BRST transformation

$$
\begin{array}{ll}
\delta_\lambda x^i_{(a)} = -\lambda\frac{(2\pi\alpha')^2}{\epsilon}\delta_{a0}\mathcal{F}_{ij}\mathcal{C}^j, & \delta_\lambda y_1 = -\lambda\Delta_{12}\mathcal{C}^2, \\
\delta_\lambda y_2 = -\lambda\mathcal{C}^1, & \delta_\lambda\bar{\mathcal{C}}_i = -\lambda\mathcal{B}_i, \\
\delta_\lambda\mathcal{C}^i = 0, & \delta_\lambda\mathcal{B}_i = 0,
\end{array}
\tag{4.10}
$$

where λ is an infinitesimal Grassmann valued parameter.

V. CONCLUSIONS

In conclusion, we have constructed the first-class Hamiltonian for the D-brane systems with constant NS B-field in the Batalin-Fradkin-Tyutin scheme. In the BRST-BFV formalism, we have next introduced canonical sets of ghosts and anti-ghosts together with auxiliary fields and thus we have constructed their BRST invariant first-class Hamiltonian and the corresponding effective Lagrangian which is invariant under transformation with nilpotent BRST charge. Moreover it has been shown that the presence of auxiliary variables introduced via the improved Dirac formalism plays a crucial role in the construction of the BRST invariant Lagrangian, since the BRST-BFV formalism is applicable to theories with the first-class constraints associated with these auxiliary variables.

It would be desirable if the continuum limit of our discretized BRST invariant effective Lagrangian can be formulated. This work is in progress and will be reported elsewhere.

Acknowledgments

[1] A. Connes, M.R. Douglas, A. Schwarz, JHEP 02 (1998) 003.
[2] M.R. Douglas, C. Hull, JHEP 02 (1998) 008.
[3] N. Seiberg, Phys. Rev. Lett. 79 (1997) 3577.
[4] D. Bigatti, L. Susskind, Phys. Rev. D 62 (2000) 066004.
[5] N. Seiberg, E. Witten, JHEP 09 (1999) 032.
[6] P.A.M. Dirac, *Lectures on Quantum Mechanics* (Yeshiba University Press, New York, 1964).
[7] S.T. Hong, Y.J. Park, Phys. Rep. 358 (2002) 143, and references therein.
[8] I.A. Batalin, E.S. Fradkin, Nucl. Phys. B 279 (1987) 514; I.A. Batalin, I.V. Tyutin, Int. J. Mod. Phys. A 6 (1991) 3255.
[9] C.S. Chu, P.M. Ho, Nucl. Phys. B 550 (1999) 151; T. Lee, Phys. Rev. D 62 (2000) 024022; C.S. Chu, P.M. Ho, Nucl. Phys. B 568 (2000) 447.
[10] S.T. Hong, W.T. Kim, Y.J. Park, M.S. Yoon, Phys. Rev. D 62 (2000) 085010.
[11] C.S. Chu, P.M. Ho, Nucl. Phys. B 636 (2002) 141.

[12] V.O. Rivelles, Phys. Lett. B 558 (2003) 191.

[13] O.F. Dayi, Phys. Lett. B 560 (2003) 239.

[14] M.B. Cantcheff, P. Minces, Phys. Lett. B 557 (2003) 283.

[15] C. Becchi, A. Rouet, R. Stora, Ann. Phys. 98 (1976) 287; I.V. Tyutin, Lebedev Preprint 39 (1975) unpublished.

[16] E.S. Fradkin, G.A. Vilkovisky, Phys. Lett. B 55 (1975) 224; M. Henneaux, Phys. Rep. 126 (1985) 1.